If $(3,2,5)$ is a point on the surface $F(x,y,z)=0$ and $\nabla F(3,2,5) = \langle 1,0,-7\rangle$, how can we characterize the vectors parallel to the tangent plane of the surface @ $(3,2,5)$?

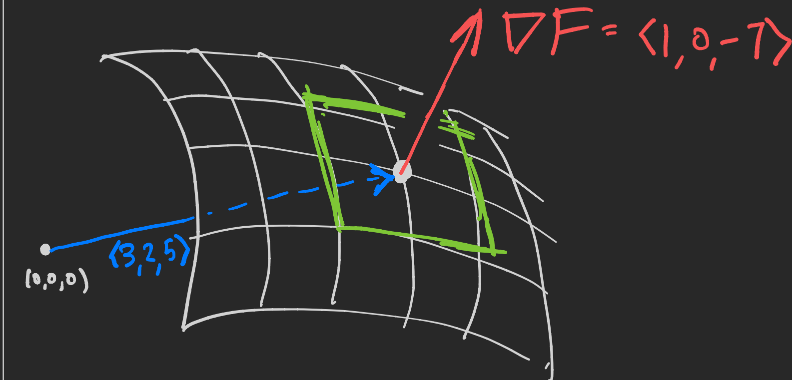
Rmk the tangent plane in question is

$$\vec{n} \cdot (\vec{r} - \vec{r}_0) = 0$$

a normal vec; point on plane

$$\langle 1,0,-7\rangle \cdot \langle x-3, y-2, z-5\rangle = 0$$

$$(x-3) - 7(z-5) = 0.$$

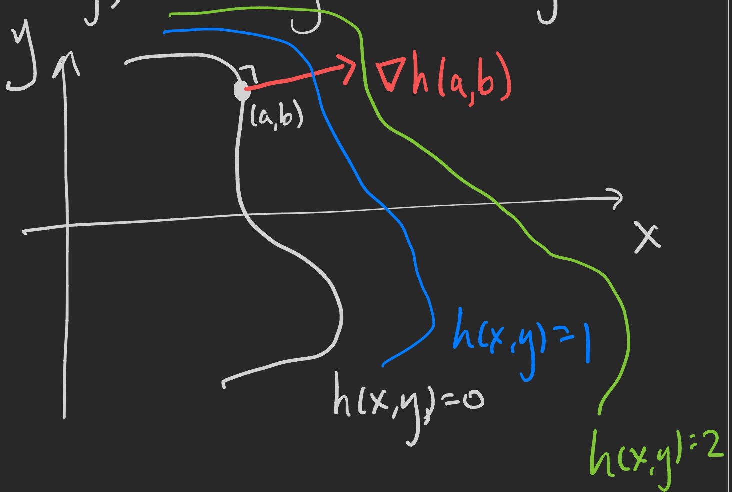


$F(x,y,z)=0$

To check if $\vec{u}$ (some vec.) is parallel to ▱, need to check $\vec{u} \cdot \nabla F = 0$.

e.g. $\langle 14,-6,2\rangle \cdot \langle 1,0,-7\rangle = 14+0-14 = 0$

Similarly, if dealing with $h(x,y)=0$



Why orthog? Recall:

(1) $\quad D_{\vec{u}} f(a,b) = \nabla f(a,b) \cdot \vec{u}$

(2) if $\vec{u}$ is parallel to the level set @ (a,b), then $D_{\vec{u}} f(a,b) = 0$.

(1)+(2) $\Rightarrow$ the gradient ∇f is orthogonal to all vectors parallel to the level set (i.e. the gradient is orthogonal to the level set)

<u>Rmk</u> If $f(x,y,z)$ is a function taking 3 inputs and giving 1 output:

The graph is a 3-dimensional object in $(3+1)$-dimensional space

Its level sets are $(3-1)$-dimensional objects (usually...) in 3-dim space.